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Uncertainty quantication of critical speed for railway vehicle dynamics

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Uncertainty quantification of critical speed for railway vehicle dynamics

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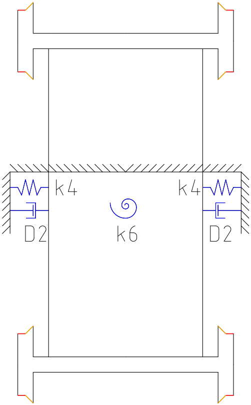
Railway vehicle dynamics - Euler's formulation[1]

$$m\ddot{\vec{x}}_1 + 2D_2\dot{\vec{x}}_1 + 2k_4\vec{x}_1 + 2F_X(\xi_{x1},\xi_{y1}) + 2F_X(\xi_{x2},\xi_{y2}) = 0$$

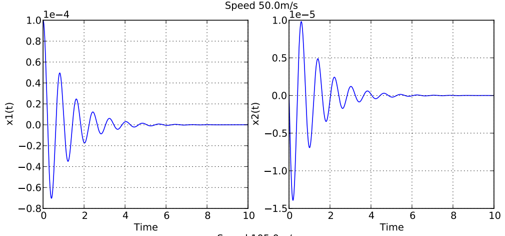
$$I\ddot{\vec{x}}_2 + k_6\vec{x}_2 + 2ha\left[F_X(\xi_{x1},\xi_{y1}) - F_X(\xi_{x2},\xi_{y2})\right] + \\ + a\left[F_Y(\xi_{x1},\xi_{y1}) + F_Y(\xi_{x2},\xi_{y2})\right] = 0$$

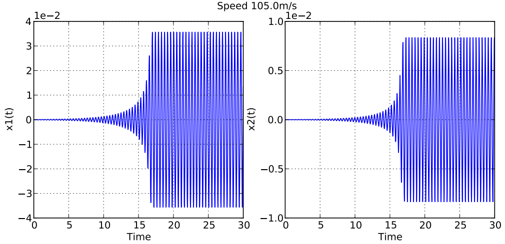
where F_X and F_Y are the creep forces, and determine a non-linear coupling of $\vec{x}_1$ and $\vec{x}_2$. Among other components, these forces involve also the running velocity v of the vehicle, the conicity of the wheels and the wheel-rail friction.

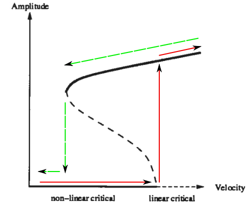


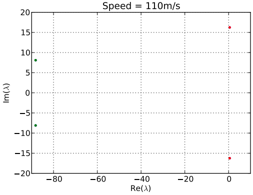
[1] H.True and C.Kaas-Petersen 1983

Railway vehicle dynamics - Hunting





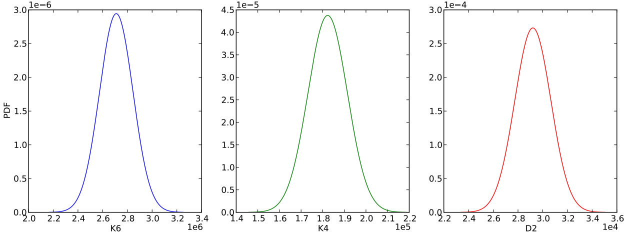




Railway vehicle dynamics - Stochastic Model

Let's now assume that the suspension components k_6, k_4 and D_2 are known within a certain level of accuracy and model this by:

$$k_6 \sim \mathcal{N}(3.44 \cdot 10^6, 2.96 \cdot 10^{10}), \quad \text{(std. of approx. 5\%)}$$
$$k_4 \sim \mathcal{N}(9.12 \cdot 10^4, 4.15 \cdot 10^7), \quad \text{(std. of approx. 7\%)}$$
$$D_2 \sim \mathcal{N}(1.46 \cdot 10^4, 1.07 \cdot 10^6), \quad \text{(std. of approx. 7\%)}$$



What are the dynamics of the system under these conditions?

Uncertainty Quantification - Traditional Approaches

Analytical Methods

- Moment Equation
- Perturbation Method

Pros.: recover the exact solution
Cons.: problem-dependent, cumbersome

Sampling Methods

- (MC) Monte Carlo – $\mathcal{O}\left(N^{-1/2}\right)$
- (QMC) Quasi Monte Carlo – $\mathcal{O}\left((\log N)^d/\sqrt{N}\right)$
- (MCMC) Markov Chain Monte Carlo

Pros.: general applicability, MC convergence independent from dimensionality d
Cons.: very slow convergence

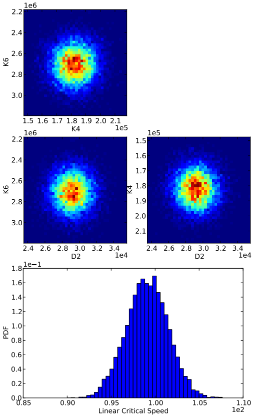


Figure: Linear critical speed distribution using 10^4 realizations for MC method.

UQ - Generalized Polynomial Chaos (gPC)[2]

Let Y be a r.v. with CDF $F_Y(y)$. Use the Nth-degree gPC expansion of the random parameters and the solution

$$Y_N = \sum_{k=0}^{N} \hat{a}_k \Phi_k(Z), \quad \hat{a}_k = \frac{1}{\gamma_k} \int_{I_Z} F_Y^{-1}(F_Z(z)) \Phi_k(z) dF_Z(z)$$

$$u_N(t, Z) = \sum_{k=0}^{N} \hat{u}_k(t) \Phi_k(Z)$$

$$\begin{cases} \mathbb{E}\left[\partial_t u_N(t, Z) \Phi_k(Z)\right] = \mathbb{E}\left[f(u_N) \Phi_k(Z)\right], & D \times (0, T] \\ \hat{u}_k(0) = \frac{1}{\gamma_k} \mathbb{E}\left[u(0, Z) \Phi_k(Z)\right], & D \times \{t = 0\} \end{cases}$$

$$\mu_u(t) \approx \mathbb{E}\left[u_N(t, Z)\right] = \hat{u}_0(t)$$

$$\mathbf{Var}\left[u(t, Z)\right] \approx \mathbf{Var}\left[u_N(t, Z)\right] = \sum_{k=1}^{N} \gamma_k \hat{u}_k^2(x, t)$$

where $\mathbb{E}\left[f(Z)\right] = \int_{I_Z} f(z) dF_Z(z)$ and $\{\phi_i(Z)\}_{i=0}^{N}$ are proper orthonormal basis.

[2] D.Xiu and G.Karniadakis 2004

UQ - gPC on Railway Vehicle Dynamics

The N-th order gPC expansion of the problem is given by

$$\begin{cases} \mathbb{E}\left[\partial_t u_{1,N}\phi_k\right] = & \mathbb{E}\left[u_{2,N}\phi_k\right] \\ \mathbb{E}\left[\partial_t u_{2,N}\phi_k\right] = & -2\mathbb{E}\left[D_{2,N}u_{2,N}\phi_k\right] - 2\mathbb{E}\left[k_{4,N}u_{1,N}\phi_k\right] \\ & -2\mathbb{E}\left[\left(F_X(\xi_{x1},\xi_{y1}) + F_X(\xi_{x2},\xi_{y2})\right)\phi_k\right] \\ \mathbb{E}\left[\partial_t u_{3,N}\phi_k\right] = & \mathbb{E}\left[u_{4,N}\phi_k\right] \\ \mathbb{E}\left[\partial_t u_{4,N}\phi_k\right] = & -\mathbb{E}\left[k_{6,N}u_{3,N}\phi_k\right] - 2ha\mathbb{E}\left[\left(F_X(\xi_{x1},\xi_{y1}) - F_X(\xi_{x2},\xi_{y2})\right)\phi_k\right] \\ & -a\mathbb{E}\left[\left(F_Y(\xi_{x1},\xi_{y1}) + F_Y(\xi_{x2},\xi_{y2})\right)\phi_k\right] \end{cases}$$

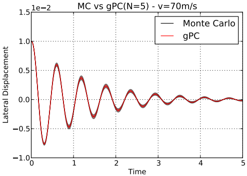
where k is a multi index such that

$$u_{i,N}(t,Z) = \sum_{|k|\leq N} \hat{u}_k(t)\Phi_k(Z), \qquad i = 1,\dots,4$$

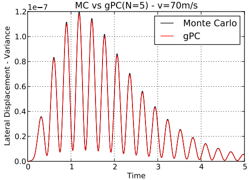
We obtain a system of $K = \sum_{i=0}^{N} \binom{i+(d-1)}{(d-1)}$ coupled equations that can be treated using standard ODE solvers. The following table shows how this number scales:

N	1	2	3	4	5	6	7
$d=1$	2	3	4	5	6	7	8
$d=2$	3	6	10	15	21	28	36
$d=3$	4	10	20	35	56	84	120
$d=4$	5	15	35	70	126	210	330
$d=5$	6	21	56	126	252	462	792

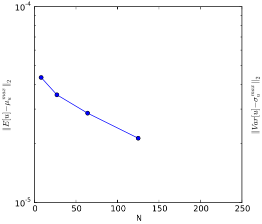
UQ - gPC on Railway Vehicle Dynamics

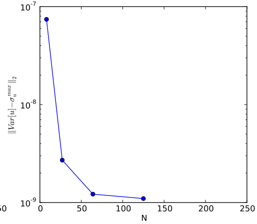


(a) Mean and variance



(b) Variance





Pros: Elegant fomulation, one single solution of the system, optimal accuracy

Cons: Intrusive and cumbersome to implement, non-linearities must be treated carefully, weak on time-dependent problems (but there exist improvements).

UQ - Probabilistic Collocation Methods (PCM)

Solve the deterministic ODE on a "proper" set $\Theta_M = \left\{Z^{(j)}\right\}_{j=1}^{M}$ of nodes in the random space:

$$\begin{cases} \partial_t u(t, Z^{(j)}) = f(u), & D \times (0, T] \\ u(0) = u_0, & D \times \{t = 0\} \end{cases}$$

This will give $u^{(j)} = u(t, Z^{(j)})$ solutions on which we can apply interpolation rules or projection rules. Let's consider the discrete projection:

$$u_N(Z) = \sum_{|k| \leq N} \hat{u}_k(t) \Phi_k(Z)$$

$$\hat{u}_k(t) = \frac{1}{\gamma_k} \mathbb{E}\left[u(t, Z)\phi_k(Z)\right] = \frac{1}{\gamma_k} \int u(z)\phi_k(z) dF_Z(z)$$

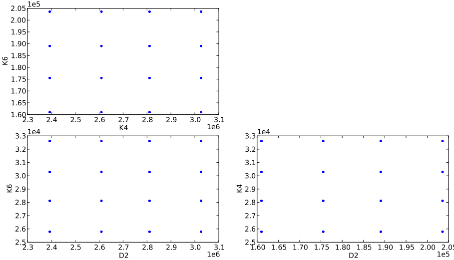
where the integral can be computed by cubature rules using the "properly" selected set of nodes Θ_M. Then statistics can be easily obtained:

$$\mu_u(t) \approx \mathbb{E}\left[u_N(t, Z)\right] = \hat{u}_0(t)$$

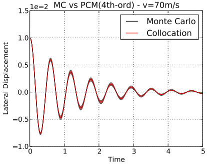
$$\mathbf{Var}\left[u(t, Z)\right] \approx \mathbf{Var}\left[u_N(t, Z)\right] = \sum_{|k| \leq N} \gamma_k \hat{u}_k^2(x, t)$$

Target: obtain the "best" statistics out of the smallest number of simulation!

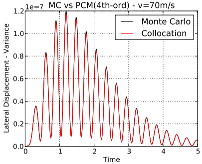
UQ - PCM on Railway Vehicle Dynamics



(c) Collocation points, $N = 3$



(d) Mean and variance



(e) Variance

Figure: PCM vs. Monte Carlo

Hermite polynomials are chosen as basis for the projection/cubature. Projection with these polynomials can be highly accurate, using proper Gauss quadrature nodes and weights, for which analytical formulas exist.

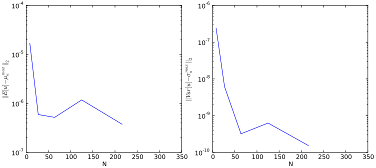
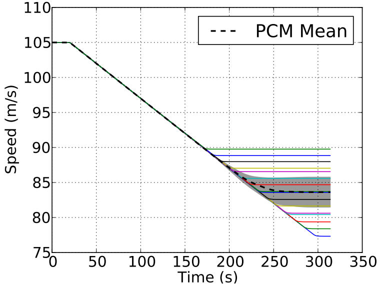


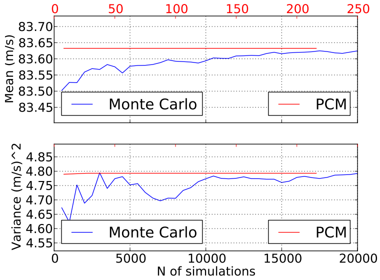
Figure: PCM convergence to highest accuracy (mean and variance).

UQ - PCM for Critical Speed statistics

Let's extend the dynamical system in order to obtain a "controlled" ramping method:

$$
\begin{cases}
\dot{\vec{u}}_1 = & \vec{u}_2 \\
m\dot{\vec{u}}_2 = & -2D_2\vec{u}_2 - 2k_4\vec{u}_1 - 2F_X(\xi_{x1},\xi_{y1}) - 2F_X(\xi_{x2},\xi_{y2}) \\
\dot{\vec{u}}_3 = & \vec{u}_4 \\
I\dot{\vec{u}}_4 = & -k_6\vec{u}_3 - 2ha\left[F_X(\xi_{x1},\xi_{y1}) - F_X(\xi_{x2},\xi_{y2})\right] - \\
 & -a\left[F_Y(\xi_{x1},\xi_{y1}) + F_Y(\xi_{x2},\xi_{y2})\right] \\
\dot{\vec{v}} = & \begin{cases} 0 & \text{if } t < t_{st} \vee \|\vec{u}\|_2 < \varepsilon_{min} \\ -\|\vec{u}\|_2 & \text{if } \|\vec{u}\|_2 < \varepsilon_{max} \\ -\varepsilon_{max} & \text{otherwise} \end{cases}
\end{cases}
$$





Outlook - Future work

UQ on **Railway vehicle dynamics**

- Uncertainty quantification with Sparse Grids
- Uncertainty quantification on a realistic model
- Parameter space compression and compressed sensing

UQ on **Free Water Wave Dynamics**

- Parametrization of random fields

Other applications of Uncertainty Quantification